

Mortality forecasting using logistics equation

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Problem

Well known methods for forecasting usually take into account the “internal” information obtained from historical as well as from experimental data. Assumptions being based on analysis of tendency with time become incorrect due to:

- Appearing of the new factors which can break the tendency;**
- Changing influence of some parameters with time due to non-linear solution;**
- etc.**

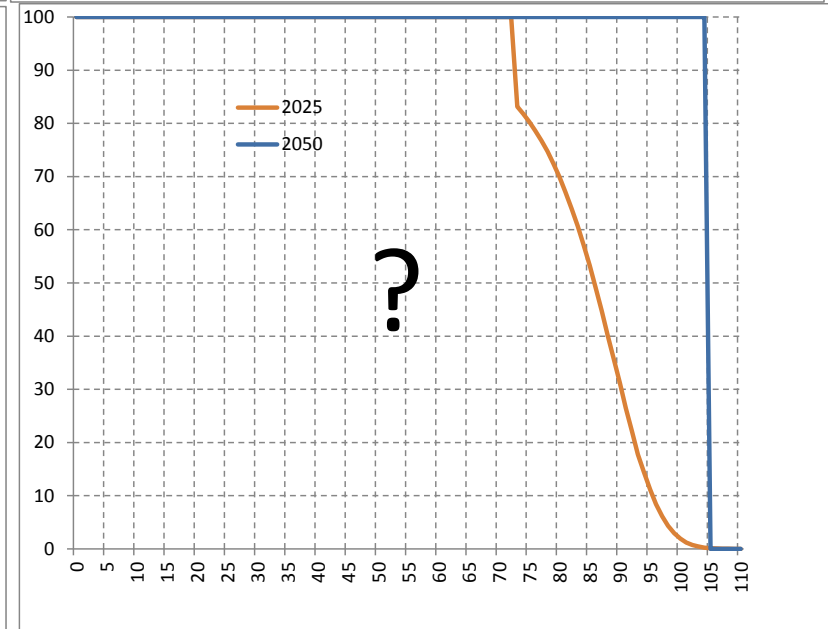
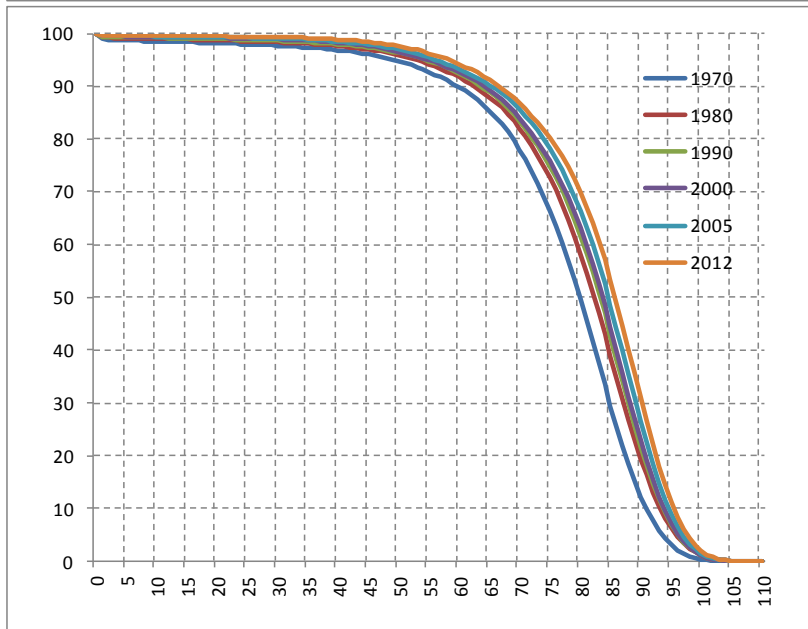
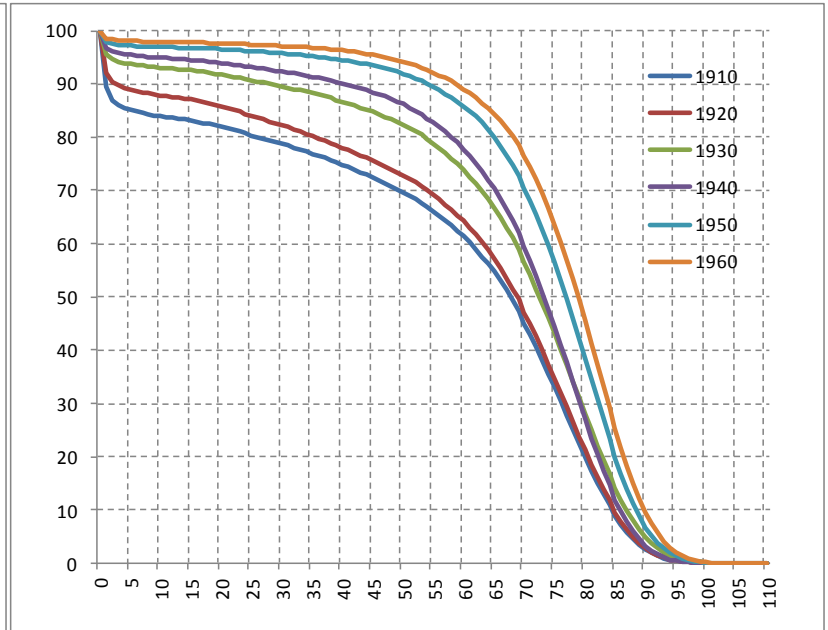
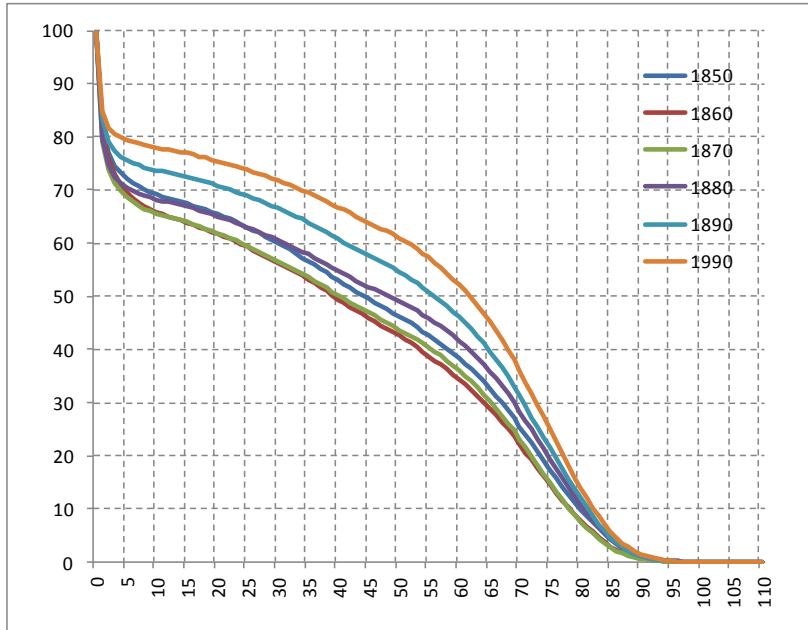
Assumptions “If then” scenario (this notorious “IF”)

A **laconic phrase** is a very concise or terse statement, named after Laconia.

If you would open the gate you will be alive – the offer of Alexander the Great to citizens of Laconia.

The answer of Laconia - IF

Historical data lx (Holland, Females)

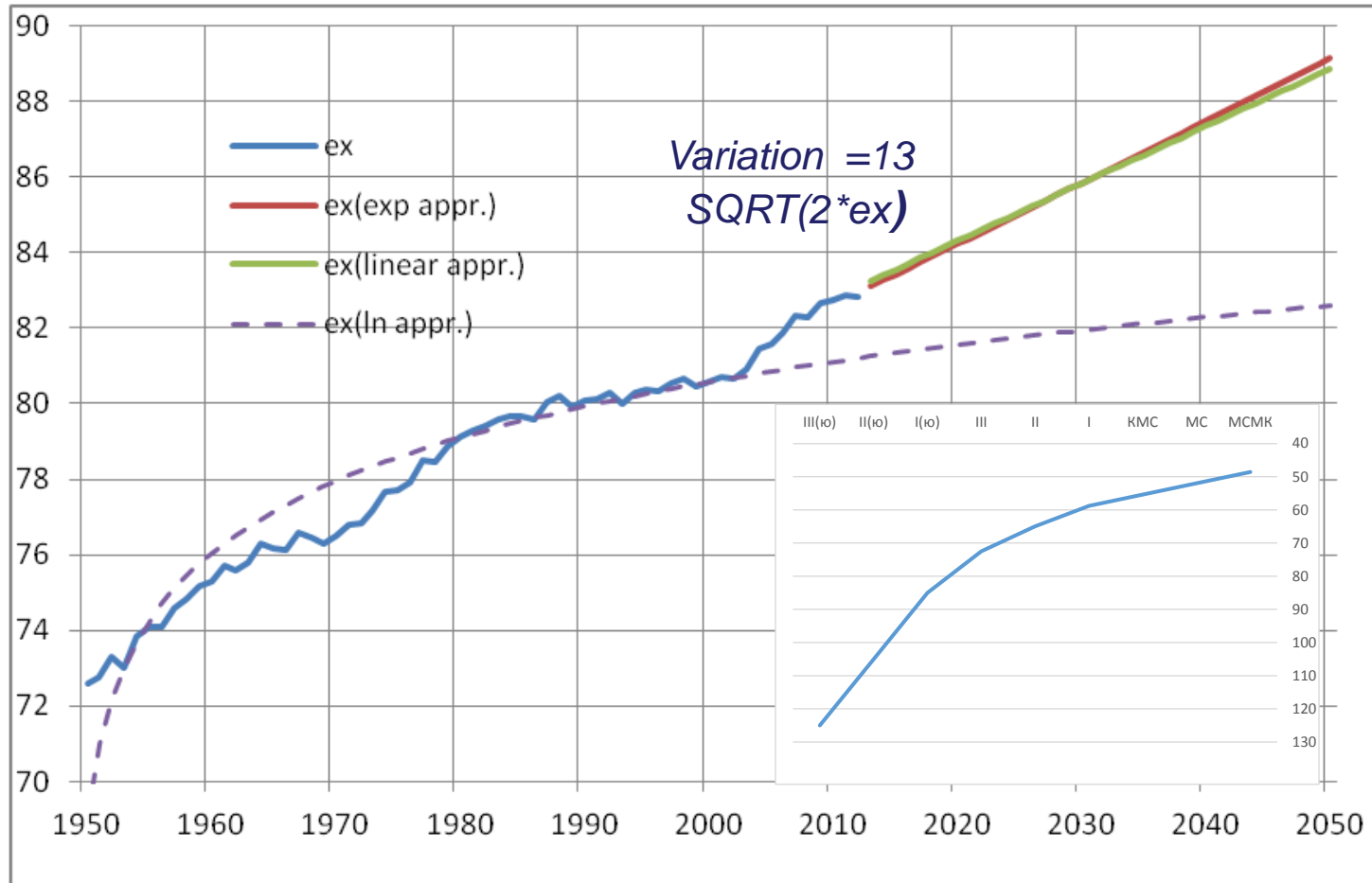


Idea

Why not to search the analogue in another areas of our knowledge? Let us try to find the similar shape of the survival function among well-known solutions and models. For example, one can see that solution of logistics equation has similar shape as survival function. Is it possible playing with coefficients of logistics equation to close its solution to the shape of survival?

Using as example historical data dynamics for females of Holland population (www.mortality.org) coefficients of logistics equation were found for criterion function and forecast of life expectancy was built.

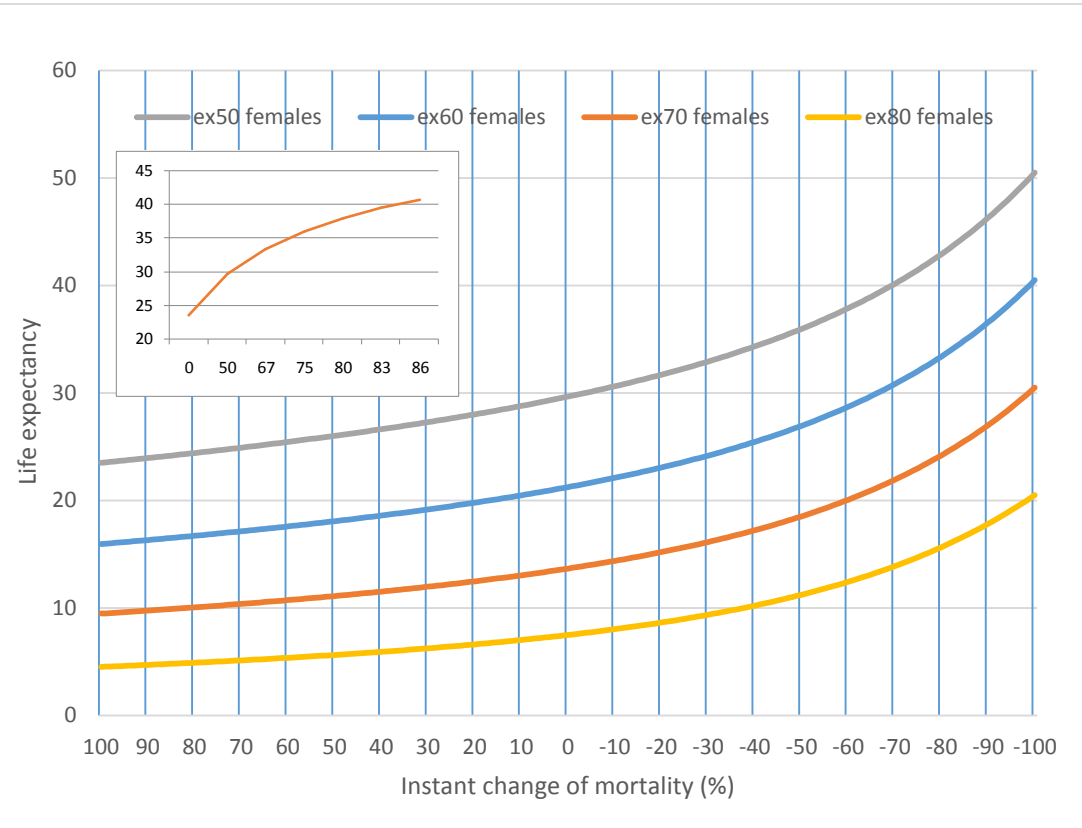
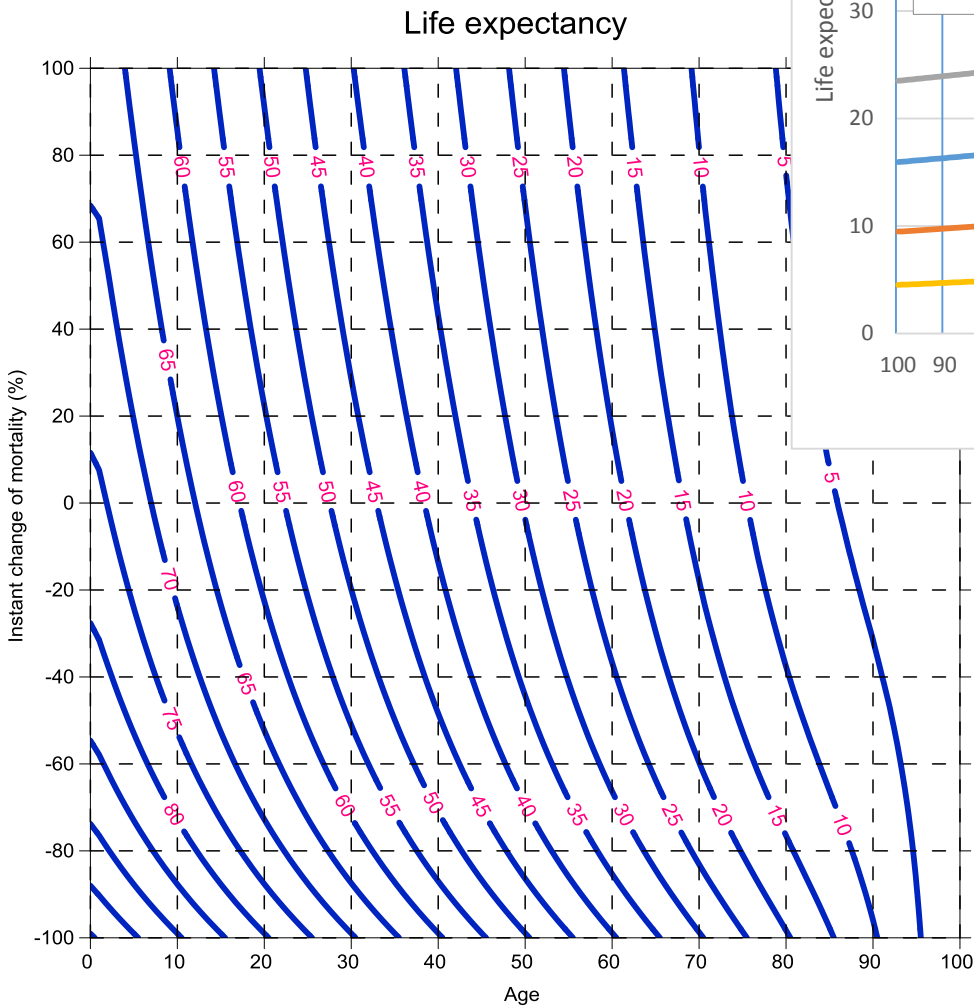
Forecast of e_0 , using trend's models (females, Holland)



Sport's example: To swim 100 m per 1 min takes 3-4 years, per 55 sec takes more 5 years. How much time is necessary to swim 100 m per 45 sec?

Answer: The world record for 50 m pool is 46.91

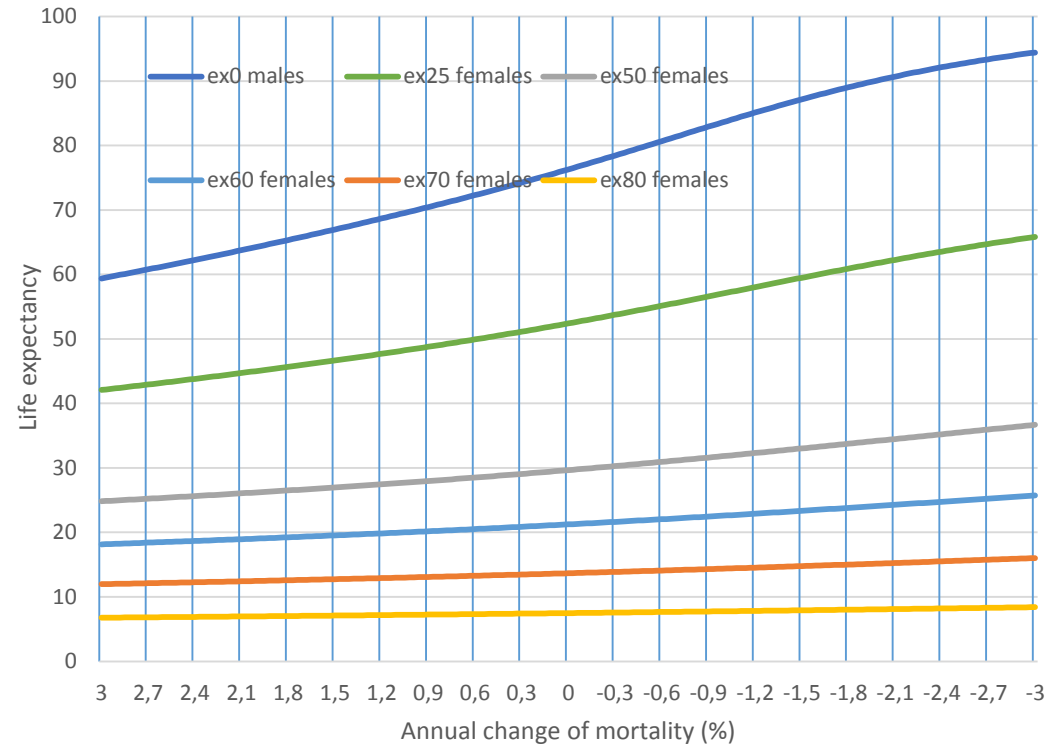
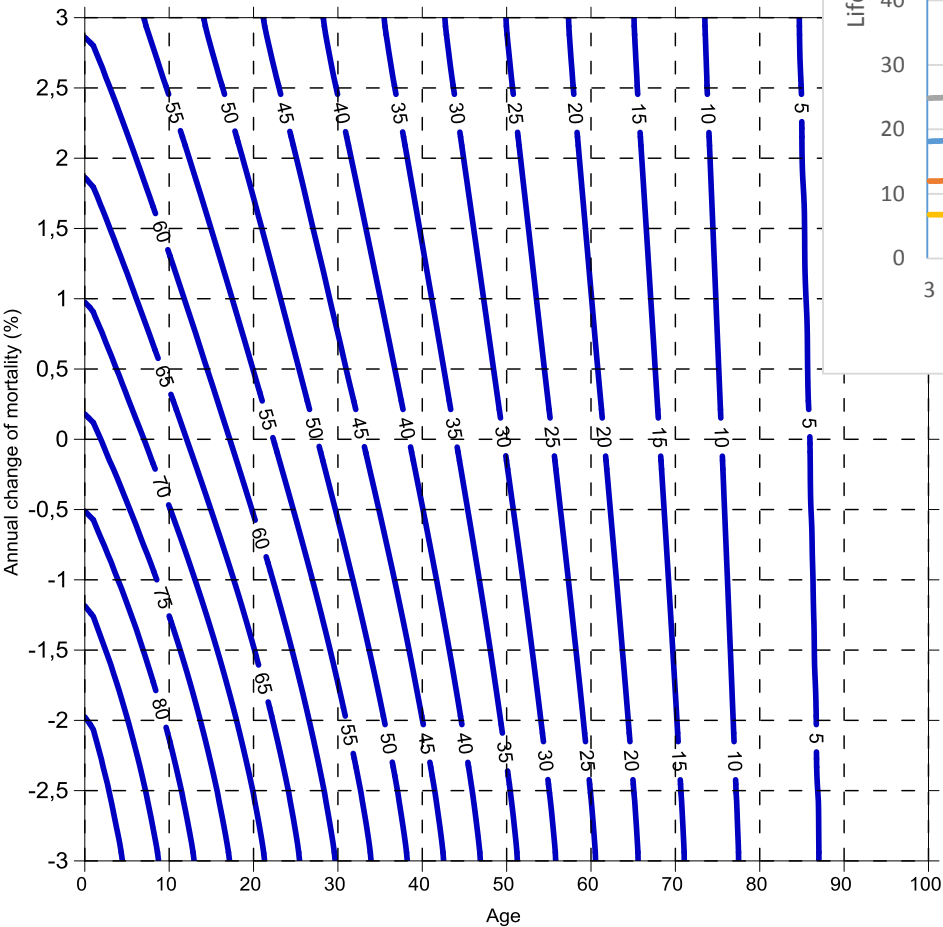
Does the limit of life expectancy exist? How long could be continued effect of mortality improvement?



ex	%	times
23,5		
29,7	50	2
33,3	67	3
36,0	75	4
37,9	80	5
39,4	83	6
40,7	86	7

Consequences of annual changes of mortality rate in old ages are more real.

Life expectancy



But for young ages it looks like a challenge.

Survival function & logistics equation

$$\frac{dS}{dt} = -k * S$$

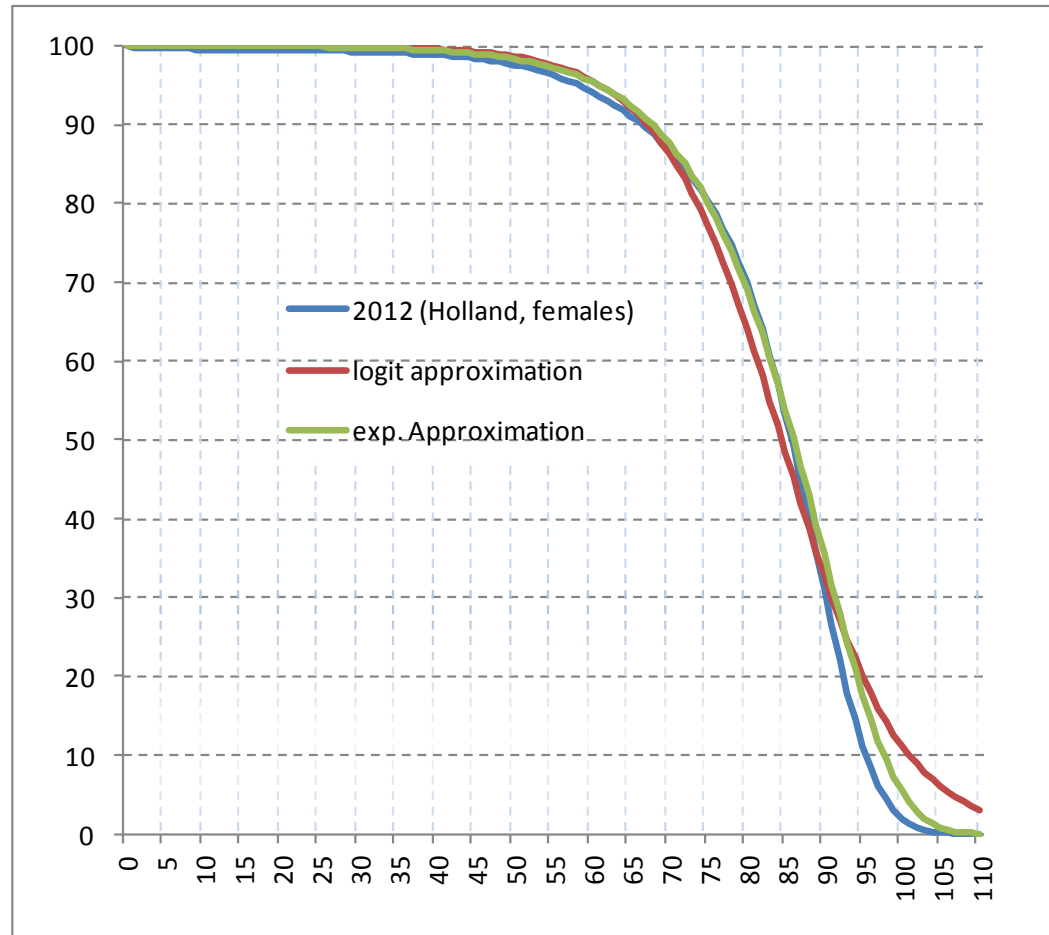
$$k = k_0 * \exp(a * x)$$

$$S = S_0 * \exp\left[\frac{k_0}{a} * (1 - \exp(a * x))\right]$$

$$\frac{dD}{dx} = a * D - b * D^2,$$

D - number of died

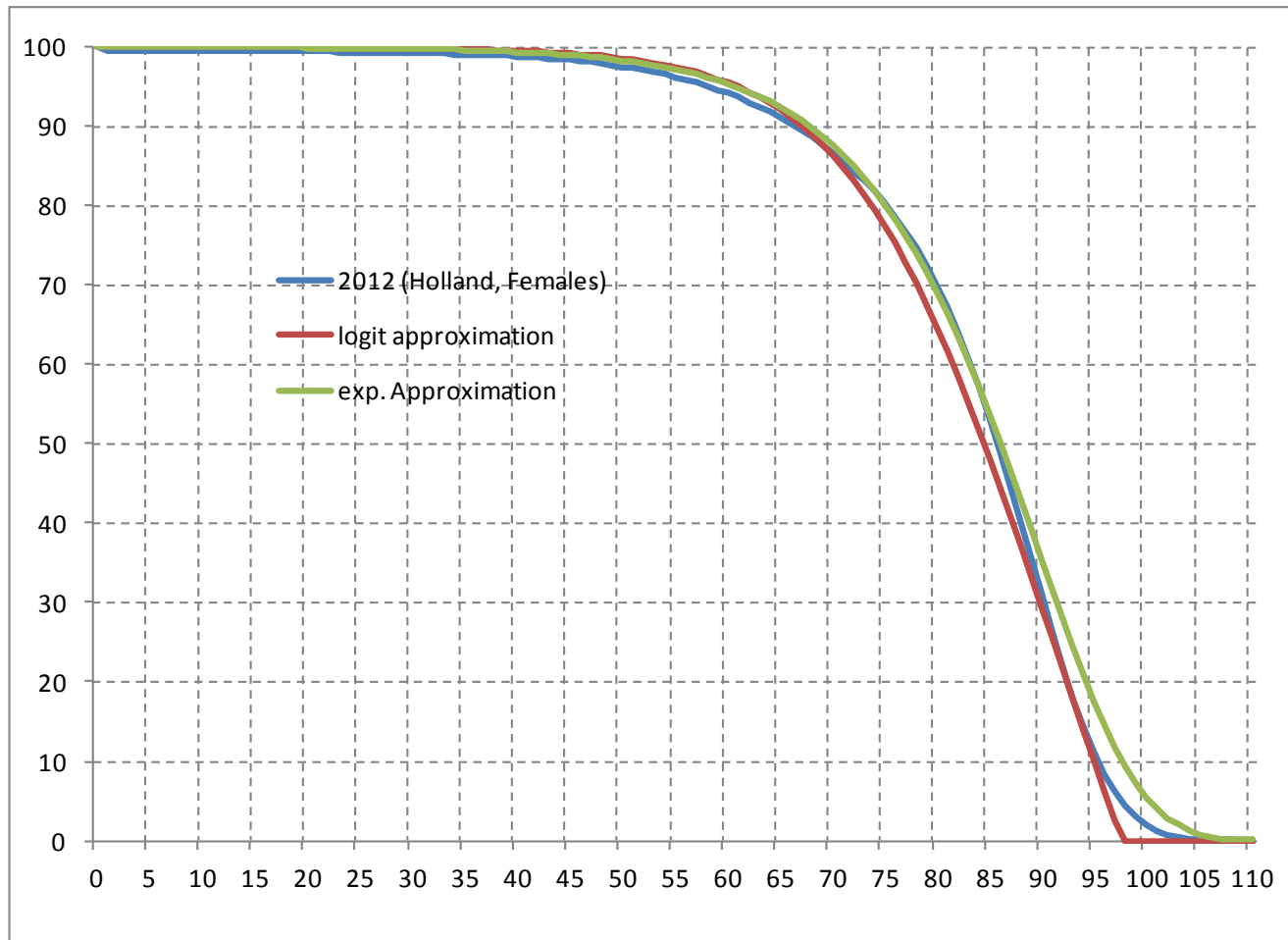
$$D = \frac{a * D_0 * \exp(a * x)}{a - b * D_0 + b * D_0 * \exp(a * x)}$$



Modified logistics equation

To eliminate the “tail” effect one more parameter was added.

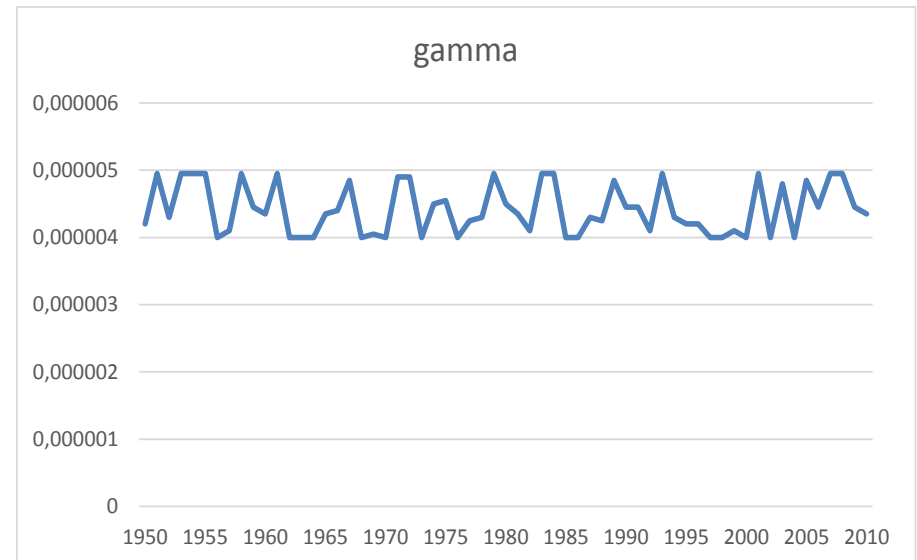
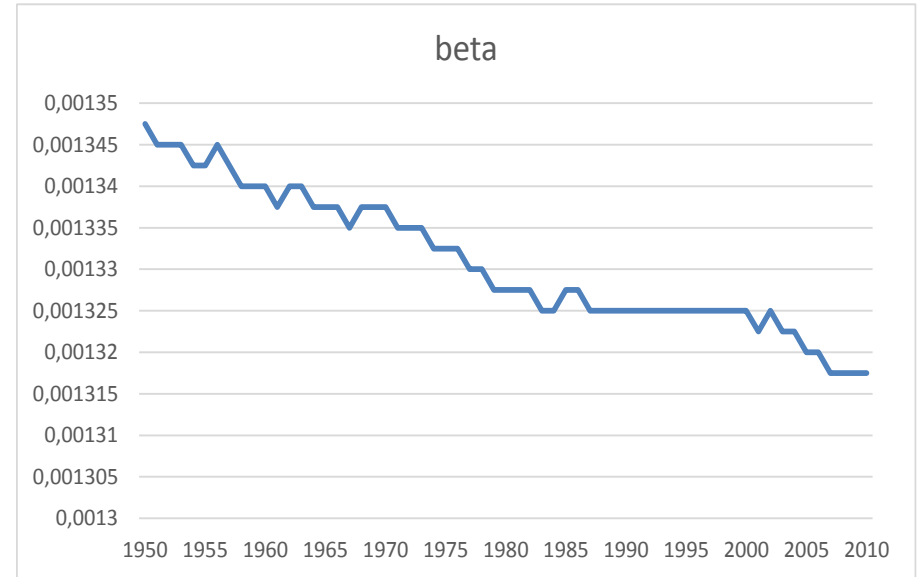
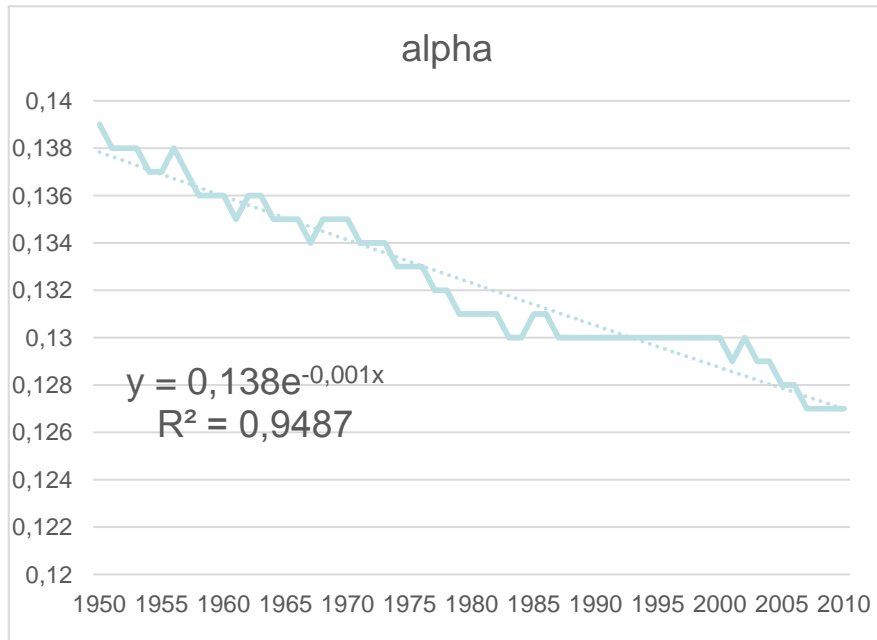
$$\frac{dD}{dx} = a * D - b * D^2 + c * D^3$$



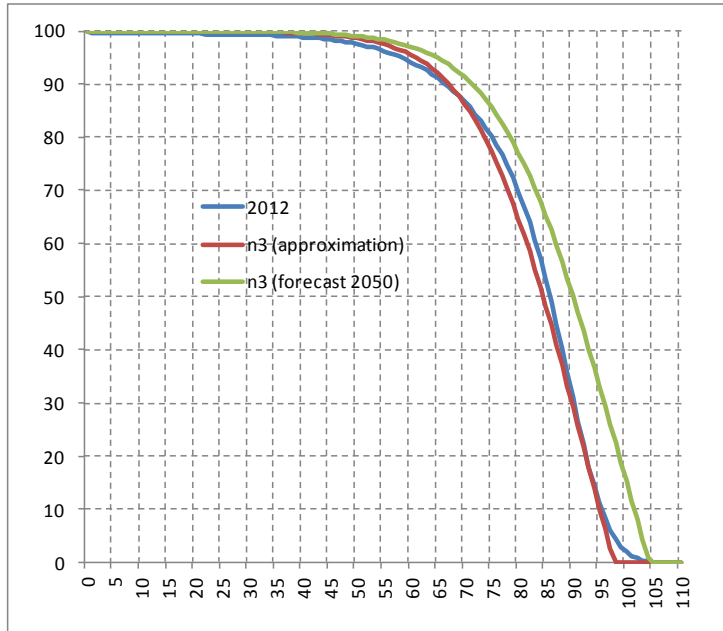
Numerical results

$$\frac{dD}{dx} = \alpha(t) * D - \beta(t) * D^2 + \gamma(t) * D^3$$

$$\sum_{x=0}^W (e_x - e'_x) = \min$$

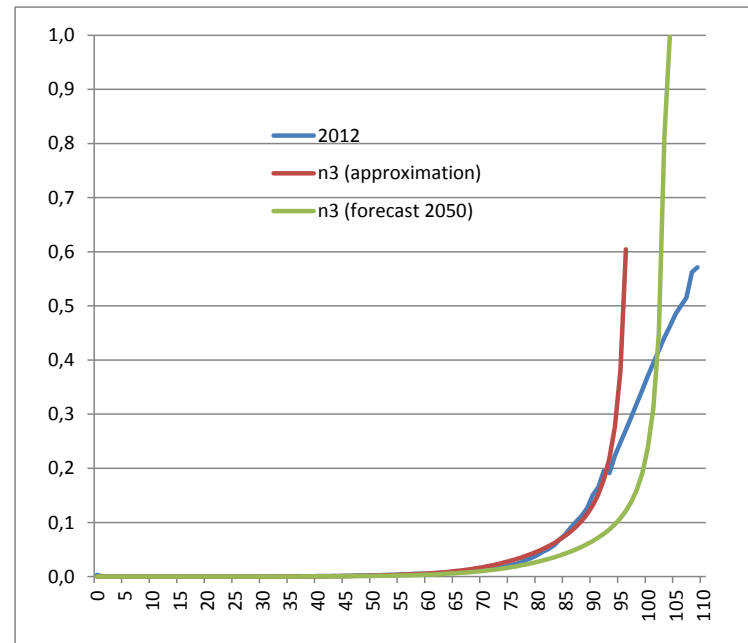
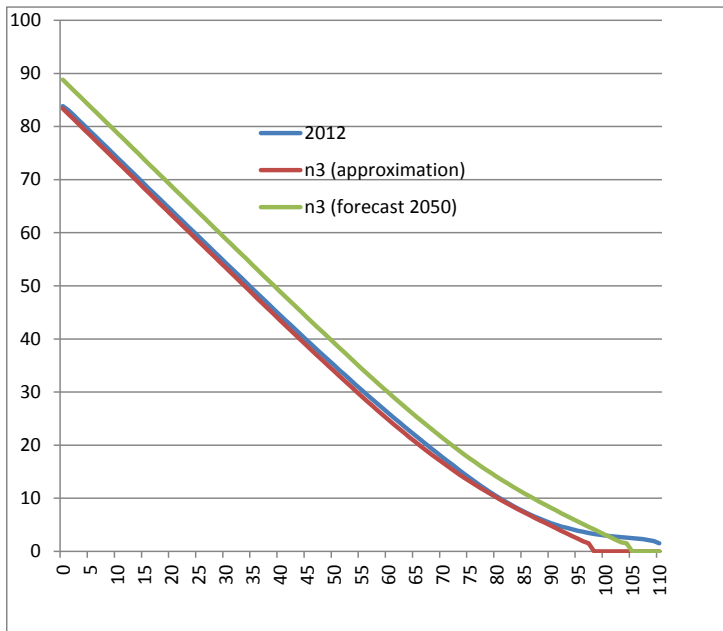


Discussing results of forecasting



Using approximation of obtained coefficients till 2050 life expectancy at birth is increasing at 8 years .

But nobody knows what will happen during this period and which new factors will appear and possibly change the situation.



Consequences/Applications

Annuities market

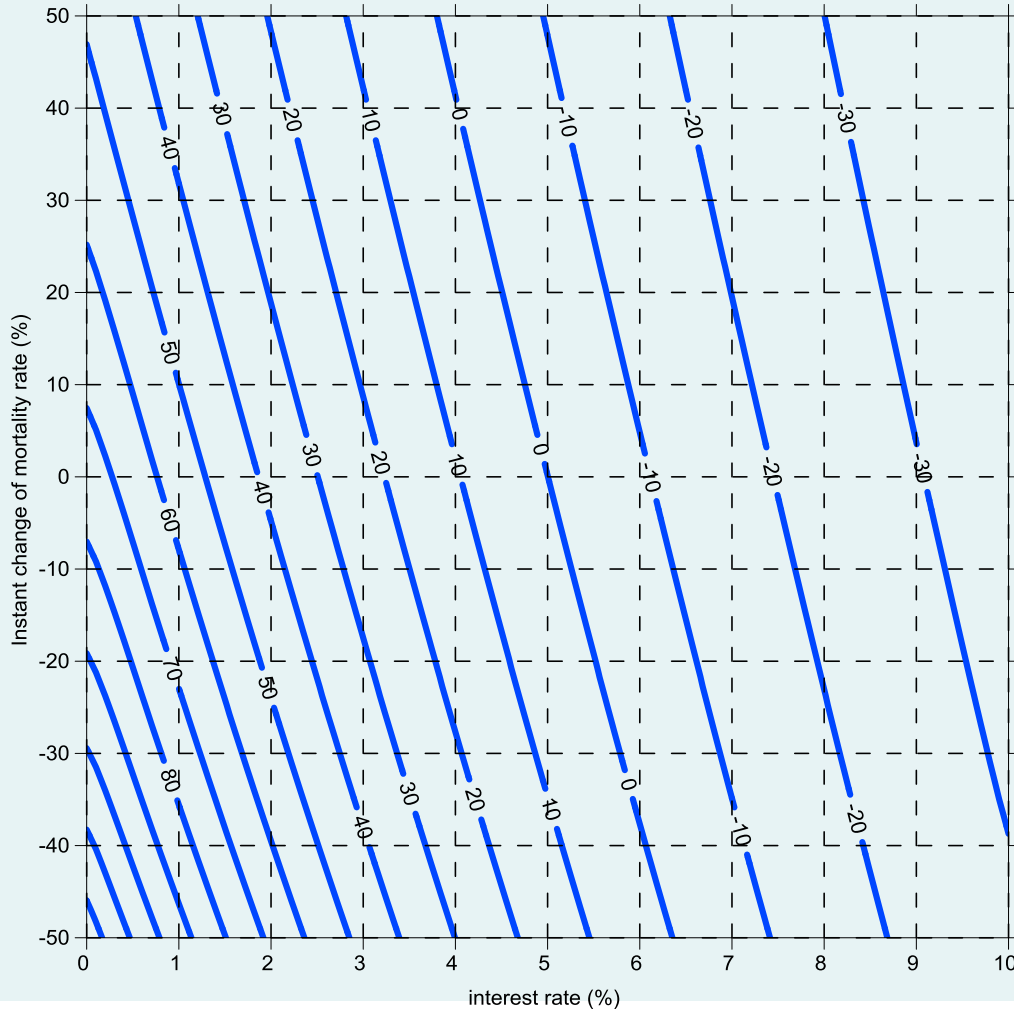
Demographic challenge for Russia

Influence on pension age for Russia

Annuities market

Change of annuity for life (%) at age 60, females

Conclusions



If initial interest rate is 5%. Too compensate mortality decreasing at 40% interest should be increased till 6%.

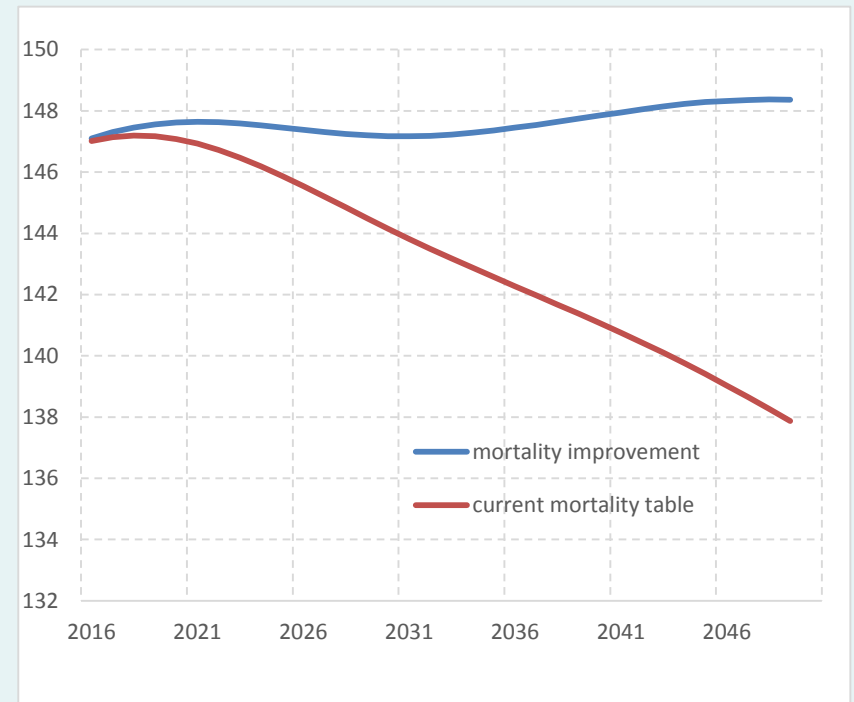
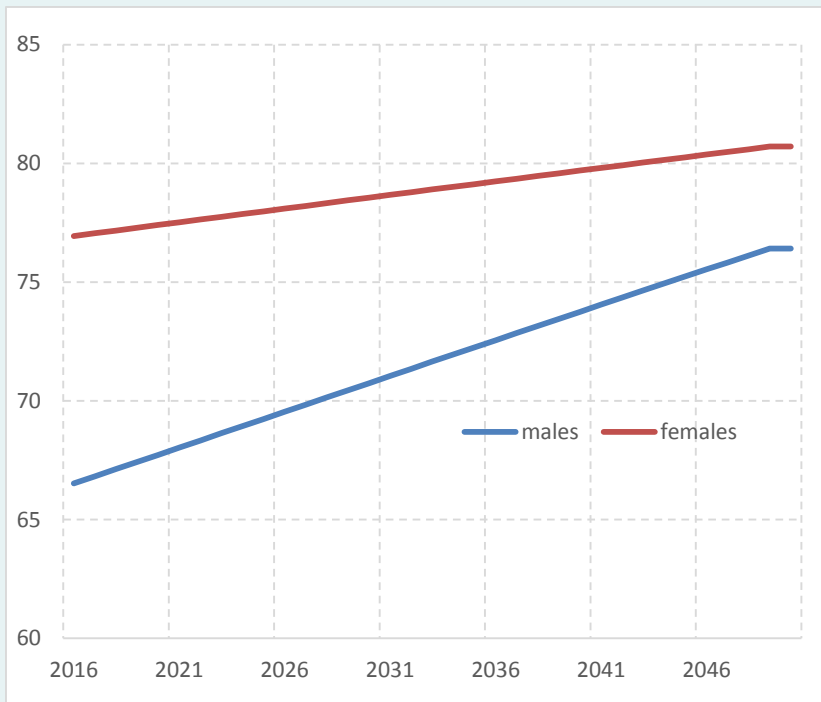
But if initial interest rate is 2%. Too compensate mortality decreasing at 40% interest should be increased till 3%.

The less interest rate the stronger influence of mortality improvement at the annuities market.

Demographic challenge (Russia)

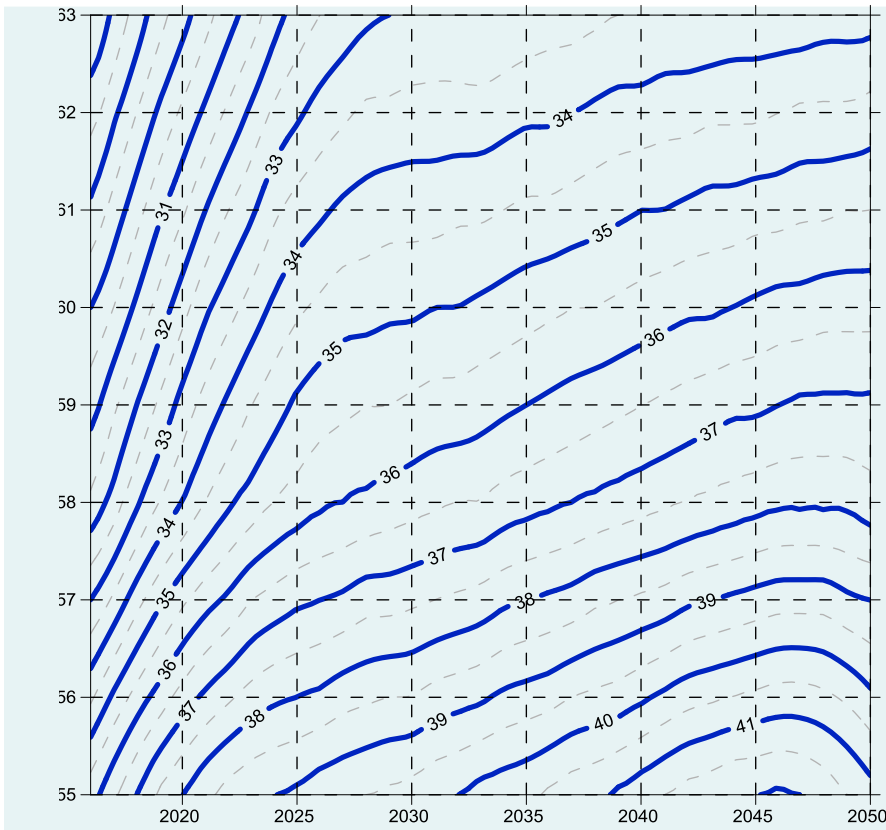
If annual decline of mortality for males would be 2% and 1% the growth of life expectancy at birth via 35 years will reach 10 years for males and 4 years for females.

It will increase the number of population approximately at 10 mln. people

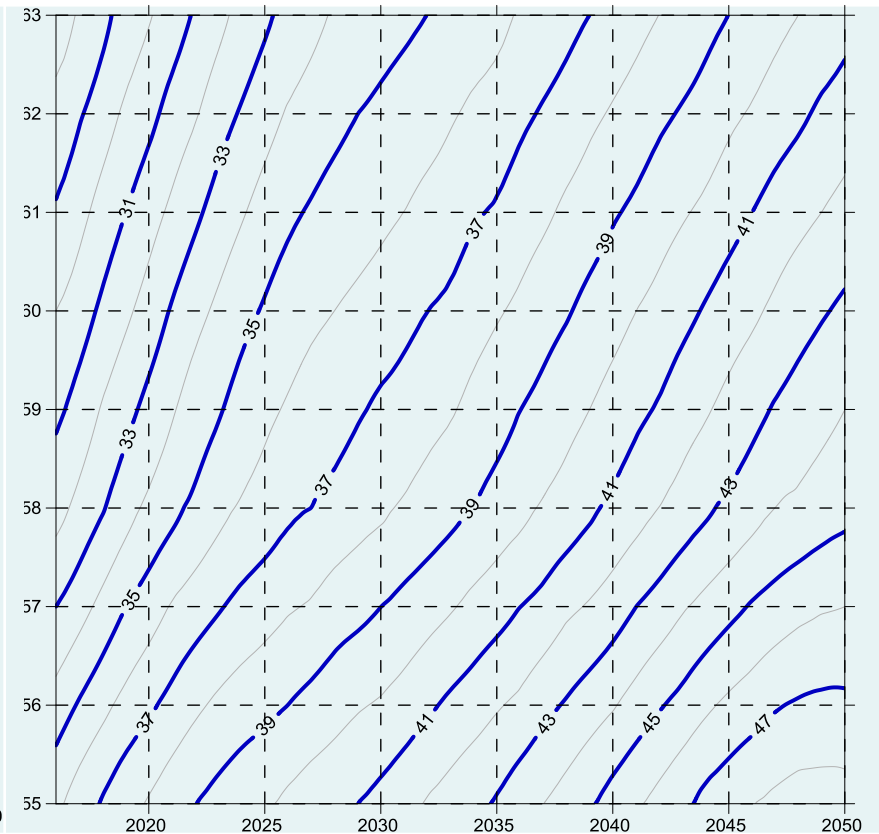


Influence on pension age (scenario for max pension age of 63 for both for males and females)

Number of pensioners depending on pension age without mortality improvement



Number of pensioners depending on pension age taking into account mortality improvement (annual decline 2% for males and 1% for females)



To keep constant number of pensioners (36 mln.) till 2035 year pension age should be increased to 63 for males and 59 for females. If mortality improvement is taken into account pension age for females should be also increased till 63 years.

Key findings

Technical (time series) forecasting of life expectancy is too uncertain. Big data analysis is too complicated. Both results are highly risky.

Assumptions are not flexible, solutions with feedbacks are possible way for future researches.

New (nature) analogues will allow to deeply understand the nature of mortality.